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Classical analogy of the concept of collective variables†

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Abstract. We discuss an analogy between certain types of scleronomic and holonomic constraints in classical mechanics, and the concept of collective degrees of freedom in quantum mechanics. We use Hamilton equations combined with the adiabatic approximation to derive expressions for the mass parameter and the potential, involving a small number of degrees of freedom, and these are compared with expressions obtained by Villars for a quantal system, using a similar approach. Finally we propose a criterion of collectivity.

1. Introduction

The objective of this paper is to see in what sense it is possible to describe the behaviour of a system with a large number of degrees of freedom by means of a small number of parameters, i.e. to suppress unnecessary or redundant degrees of freedom.

This idea is based on the assumption that intrinsic and collective motions of the many-body system are approximately decoupled. This hypothesis leads to the concept of a ‘collective’ coordinate α associated with a ‘collective path’, an effective mass $\mathcal{M}(\alpha)$, and a ‘collective’ potential $\mathcal{V}(\alpha)$, with which we construct the ‘collective’ Hamiltonian.

In our treatment we go beyond the harmonic approximation, i.e. we consider the possibility of the amplitudes becoming large, although the velocity is always kept small.

We consider a classical system, but our method is equivalent to those used for treating similar problems in quantal systems of many particles.

The equations obtained are compared with those that Villars (1977) has reached based on adiabatic time-dependent Hartree–Fock (ATDHF) theory.

The possibility of reducing the number of coordinates is suggested by the Hamilton–Jacobi theory (Goldstein 1959). Indeed, this theory enables us to define a surface in phase space characterised by two parameters α and π . This surface is such that, if at some initial time the system lies on the surface, it will remain there.

Now we say that such a surface is stationary if the system, left initially at some point in its vicinity, does not leave that vicinity. The method we develop enables us to construct such a surface in the adiabatic approximation. Our criterion provides a sufficient condition for the validity of the replacement of the actual forces by constraints. We associate this criterion with the stability of the surface. These ideas are illustrated with two simple problems in classical mechanics.

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2. Classical 'collective' Hamiltonian in the adiabatic approximation

Let us consider a system of N particles whose Hamiltonian is written as

$$H(q_i, p_i) = \frac{1}{2} \sum_{i,j=1}^N p_i p_j (M^{-1})_{ij}(q_1, \dots, q_N) + V(q_1, \dots, q_N) \quad (2.1)$$

where q_i , p_i , $M_{ij}(q)$ and $V(q)$ denote respectively the generalised coordinates, the generalised momenta, the mass parameters and the potential energy.

The Hamilton equations

$$\begin{aligned} \dot{q}_i &= \frac{\partial H}{\partial p_i} = \sum_j p_j (M^{-1})_{ij} \\ \dot{p}_i &= -\frac{\partial H}{\partial q_i} = -\frac{1}{2} \sum_{k,l} p_k p_l \frac{\partial}{\partial q_i} (M^{-1})_{kl} - \frac{\partial V}{\partial q_i} \end{aligned} \quad (2.2)$$

are derived by requiring that the action integral

$$I = \int \left(\sum_i p_i \dot{q}_i - H \right) dt$$

is stationary.

We assume that the system thus described exhibits a particular type of behaviour which can be characterised by a couple of parameters α and π . This means that there are modes of motion of the whole system for which the values taken by the q_i and p_i at successive instants of time are unambiguously determined by the values of α and π at the same instants:

$$q_i(t) = q_i(\alpha(t), \pi(t)) \quad p_i(t) = p_i(\alpha(t), \pi(t)). \quad (2.3)$$

In the adiabatic approximation (small π) we propose the parametrisation

$$q_i = g_i(\alpha) \quad p_i = \pi f_i(\alpha), \quad (2.4)$$

where π is assumed to be small.

In this hypothesis we have

$$\mathcal{H}(\alpha, \pi) = (\pi^2/2) \mathcal{M}^{-1}(\alpha) + \mathcal{V}(\alpha) \quad (2.5)$$

where π plays the role of the momentum canonically conjugate to α .

These equations allow the Lagrangian to be expressed as

$$\mathcal{L}(\alpha, \pi) = \dot{\alpha} \pi - \mathcal{H}(\alpha, \pi) \quad (2.6)$$

provided we introduce the normalisation condition

$$\sum_i f_i(\alpha) \frac{dg_i}{d\alpha} = 1 \quad (I)$$

and define the function $\mathcal{H}(\alpha, \pi)$ as

$$\mathcal{H}(\alpha, \pi) = H(g_1(\alpha), \dots, g_N(\alpha); \pi f_1(\alpha), \dots, \pi f_N(\alpha)),$$

so that from the comparison of equations (2.1) and (2.5) we obtain immediately

$$\mathcal{V}(\alpha) = V(g_1(\alpha), \dots, g_N(\alpha)) \quad (2.7)$$

and

$$\mathcal{M}^{-1}(\alpha) = \sum_{i,j=1}^N f_i(\alpha)f_j(\alpha)(M^{-1})_{ij}. \quad (2.8)$$

Equation (2.6) follows in a straightforward way from equations (2.4) and the conventional Lagrangian

$$L = \sum_i p_i \dot{q}_i - H.$$

The action principle applied to equation (2.6) leads to the equations

$$\dot{\alpha} = \partial \mathcal{H} / \partial \pi \quad \dot{\pi} = -\partial \mathcal{H} / \partial \alpha. \quad (2.9)$$

These equations describe the best time evolution compatible with the restrictions corresponding to equations (2.3).

With equations (2.5) and (2.6) we can write

$$\dot{\alpha} = \pi \mathcal{M}^{-1}(\alpha) \quad \dot{\pi} = -\frac{\pi^2}{2} \frac{d}{d\alpha} \mathcal{M}^{-1}(\alpha) - \frac{d\mathcal{V}}{d\alpha}. \quad (2.10)$$

From the equation of motion for p_i we find

$$\dot{\pi} f_i(\alpha) + \pi \dot{\alpha} \frac{df_i}{d\alpha} = -\frac{1}{2} \sum_{k,l} \pi^2 f_k(\alpha) f_l(\alpha) \frac{\partial}{\partial q_i} (M^{-1})_{kl} - \frac{\partial V}{\partial q_i}. \quad (2.11)$$

Inserting here the expressions for $\dot{\alpha}$ and $\dot{\pi}$ as given by equations (2.10) we obtain, dropping the quadratic terms in π ,

$$f_i(\alpha) \frac{d\mathcal{V}}{d\alpha} = \frac{\partial V}{\partial q_i}. \quad (II)$$

Using the equation of motion for q_i we obtain, in a similar way,

$$\frac{dg_i}{d\alpha} \dot{\alpha} = \sum_j \pi f_j(\alpha) (M^{-1})_{ij}, \quad (2.12)$$

which is equivalent to

$$\mathcal{M}^{-1}(\alpha) \frac{dg_i}{d\alpha} = \sum_j (M^{-1})_{ij} f_j(\alpha). \quad (III)$$

The self-consistent treatment of equations (I), (II) and (III) determines the functions $f_i(\alpha)$ and $g_i(\alpha)$.

These equations provide the necessary conditions in order that a system, placed at a point of the surface, at some initial time, remains there. However, they do not say anything about the stability of the surface, since such a system, represented by a point placed in the vicinity of that surface, is subject to the action of forces that can either hold it oscillating in the vicinity of the surface or push it away from this one.

The study of the stability implies the explicit consideration of the forces involved, as we shall explain in § 4.

The generalisation to higher orders in π is straightforward. We shall consider only the first-order correction. Taking into account the properties of the quantities involved under time reversal, we may write

$$q_i = g_i(\alpha) + \pi^2 h_i(\alpha) \quad p_i = \pi f_i(\alpha). \quad (2.13)$$

The canonical conjugation between α and π demands now a new normalisation condition, involving the $h_i(\alpha)$, in addition to the old one (equation (I)):

$$\sum_i \left(f_i(\alpha) \frac{dh_i}{d\alpha} - 2 \frac{df_i}{d\alpha} h_i(\alpha) \right) = 0. \quad (\text{IV})$$

The collective Hamiltonian $\mathcal{H}(\alpha, \pi)$ takes the same form as before, provided we add a new term to the expression for the collective mass:

$$\mathcal{M}^{-1}(\alpha) = \sum_{i,j} f_i(\alpha) f_j(\alpha) (\mathcal{M}^{-1})_{ij} + 2 \sum_l h_l(\alpha) \left(\frac{\partial V}{\partial q_l} \right)_{q_l = g_l(\alpha)} \quad (2.14)$$

The equation of motion for p_i gives also, besides (II), the equation

$$\frac{1}{2} f_i(\alpha) \frac{d}{d\alpha} \mathcal{M}^{-1}(\alpha) - \mathcal{M}^{-1}(\alpha) \frac{df_i}{d\alpha} = \frac{1}{2} \sum_{k,l} f_k(\alpha) f_l(\alpha) \frac{\partial}{\partial q_i} (\mathcal{M}^{-1})_{kl} + \sum_l h_l \left(\frac{\partial^2 V}{\partial q_l \partial q_i} \right)_{q_l = g_l(\alpha)} \quad (\text{V})$$

The equation of motion for q_i gives, instead of (III),

$$\mathcal{M}^{-1}(\alpha) \frac{dg_i}{d\alpha} - 2h_i(\alpha) \frac{d\mathcal{V}}{d\alpha} = \sum_{j=1}^N (\mathcal{M}^{-1})_{ij} f_j(\alpha). \quad (\text{VI})$$

The new set of conditions may be compared with equations of the type obtained by Villars using an appropriate parametrisation of the time-dependent Hartree-Fock state (Toukan and Villars 1981).

Before ending this section we shall notice the internal consistency of the equations obtained. Indeed, multiplying equation (III) by $f_i(\alpha)$, summing over the label i , and paying attention to condition (I), we obtain the equation (2.8). The same may be said for equations (IV), (V) and (VI).

3. Analogy with the Villars ATDHF theory

To study some collective aspects of the nuclear dynamics, Villars uses independent-particle wavefunctions of the type

$$\Phi(\alpha, \pi) = \exp(i\pi\hat{Q})\Phi(\alpha) \quad (3.1)$$

where $\Phi(\alpha)$ describes a static deformation and \hat{Q} is a single-particle operator even under time reversal. The state $\Phi(\alpha, \pi)$ depends on the time through $\alpha(t)$ and $\pi(t)$. To go from $\Phi(\alpha)$ to $\Phi(\alpha, \pi)$ corresponds to introducing the velocity with which the deformation develops.

The adiabatic approximation consists of assuming that the state $\Phi(\alpha, \pi)$ is not very different from $\Phi(\alpha)$ at any time, i.e. that the phase factor $\exp(i\pi\hat{Q})$ can be expanded in powers of π .

The states $\Phi(\alpha)$, the operator \hat{Q} and the time evolution of α and π are determined by the time-dependent variational principle. From this, Villars finds that α and π satisfy the canonical equations (2.9) with $\mathcal{H}(\alpha, \pi)$ given by equation (2.5) and

$$\mathcal{V}(\alpha) = \langle \Phi(\alpha) | \hat{H} | \Phi(\alpha) \rangle \quad (3.2)$$

$$\mathcal{M}^{-1}(\alpha) = \langle \Phi(\alpha) | [[i\hat{H}, \hat{Q}], i\hat{Q}] | \Phi(\alpha) \rangle, \quad (3.3)$$

provided one chooses to normalise the operator $\hat{Q}(\alpha)$ by the condition

$$\langle \Phi(\alpha) | [i\hat{P}, \hat{Q}] | \Phi(\alpha) \rangle = 1 \tag{V_1}$$

where \hat{P} is the operator of infinitesimal displacement in α defined in such a way that

$$\hat{P} | \Phi(\alpha) \rangle = i \frac{\partial}{\partial \alpha} | \Phi(\alpha) \rangle. \tag{3.4}$$

Furthermore, the variational principle gives

$$\delta \langle \Phi(\alpha) | \hat{H} - \frac{d\mathcal{V}}{d\alpha} \hat{Q} | \Phi(\alpha) \rangle = 0 \tag{V_2}$$

$$\delta \langle \Phi(\alpha) | [i\hat{H}, \hat{Q}] - \frac{\hat{P}}{\mathcal{M}(\alpha)} | \Phi(\alpha) \rangle = 0. \tag{V_3}$$

Equations (V₁), (V₂) and (V₃) determine the states $\Phi(\alpha)$ and the operator $\hat{Q}(\alpha)$.

We will show now the similarity between this set of equations and equations (I), (II) and (III) which we have obtained before. In order to do this, it is convenient first to write our equations in terms of Poisson brackets.

Let P and Q be the generator functions which are responsible for the variations of q_i and p_i with α and π respectively.

Then we must write

$$\{P, q_i\} = -\frac{\partial P}{\partial p_i} = -\frac{dg_i}{d\alpha} \tag{3.5}$$

$$\{Q, p_i\} = \frac{\partial Q}{\partial q_i} = \frac{\partial p_i}{\partial \pi} = f_i(\alpha) \tag{3.6}$$

where the classical Poisson brackets are defined in the conventional way (Goldstein 1959).

It is interesting to notice that P and Q satisfy the relation of canonically conjugate variables, i.e.

$$\{Q, P\} = \sum_{j=1}^N \frac{\partial Q}{\partial q_j} \frac{\partial P}{\partial p_j} = \sum_{j=1}^N f_j(\alpha) \frac{dg_j}{d\alpha} = 1. \tag{3.7}$$

This relation is satisfied with $p_i = 0$ and $q_i = g_i(\alpha)$, which corresponds to the calculation of the expectation value of $[i\hat{P}, \hat{Q}]$ in the state $|\Phi(\alpha)\rangle$ as is expressed by equation (V₁). So, by using the classical analogy between Poisson brackets and commutators (Dirac 1958), we prove the formal equivalence of equation (V₁) and the normalisation condition (I).

We consider now the 'collective' potential $\mathcal{V}(\alpha)$ which satisfies equation (II). By means of simple transformations we can write

$$\{H, p_i\} = \frac{d\mathcal{V}}{d\alpha} \{Q, p_i\}, \tag{3.8}$$

since the relation $\partial H / \partial q_i = \partial V / \partial q_i$ is valid when $p_i = 0$.

So we have

$$\left\{ \left(H - \frac{d\mathcal{V}}{d\alpha} Q \right), p_i \right\} = 0 \tag{3.9}$$

and, in the same way,

$$\left\{ \left(H - \frac{d\mathcal{V}}{d\alpha} Q \right), q_i \right\} = 0 \quad (3.10)$$

when $p_i = 0$ and $q_i = g_i(\alpha)$.

These equations and the properties of the Poisson brackets allow us to conclude

$$\delta \left[H - \frac{d\mathcal{V}}{d\alpha} Q \right]_{\substack{p_i=0 \\ q_i=g_i(\alpha)}} = 0 \quad (3.11)$$

where δF means

$$\delta F = \sum_i \frac{\partial F}{\partial q_i} \delta q_i + \frac{\partial F}{\partial p_i} \delta p_i.$$

The square bracket in this equation is equivalent to the calculation of the expectation value of

$$\left(\hat{H} - \frac{d\mathcal{V}}{d\alpha} \hat{Q} \right)$$

in the state $|\Phi(\alpha)\rangle$ expressed by equation (V₂).

Finally we must refer to the expressions involving the mass parameter. We write equation (III) in the form

$$\mathcal{M}^{-1}(\alpha)\{P, q_i\} = - \sum_j (\mathcal{M}^{-1})_{ij} \{Q, p_j\}. \quad (3.12)$$

Then, by means of a simple evaluation of $\{\{H, Q\}, q_i\}$, we can rewrite equation (3.12) in a very suitable way

$$\{(\{H, Q\} + \mathcal{M}^{-1}(\alpha)P), q_i\} = 0 \quad (3.13)$$

or, as previously,

$$\delta \{(\{H, Q\} + \mathcal{M}^{-1}(\alpha)P)\}_{\substack{p_i=0 \\ q_i=g_i(\alpha)}} = 0. \quad (3.14)$$

By means of the classical analogy that we have already used, it is very easy to exhibit the formal equivalence between this relation and equation (V₃) in Villar's theory.

4. Application to simple classical problems

In order to clarify the general ideas developed so far, we first consider a particle which is subject to movement on a plane, governed by the canonical equations (2.9) and the Hamiltonian

$$H = \sum_{i=1}^2 \frac{p_i^2}{2m} + \frac{K}{2} [(q_1^2 + q_2^2)^{1/2} - a]^4. \quad (4.1)$$

Changing to polar coordinates

$$q_1 = r \cos \theta \quad q_2 = r \sin \theta \quad (4.2)$$

we have

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{K}{2}(r-a)^4 \quad (4.3)$$

and

$$\dot{r} = \frac{p_r}{m} \quad \dot{p}_r = \frac{p_\theta^2}{m^2 r^3} - 2K(r-a)^3 \quad \dot{\theta} = \frac{p_\theta}{mr^2} \quad \dot{p}_\theta = 0. \quad (4.4)$$

The equations of motion for r and θ take the form

$$m\ddot{r} - \frac{p_\theta^2}{m^2 r^3} + 2K(r-a)^3 = 0 \quad (4.5)$$

$$\ddot{\theta} = 0. \quad (4.6)$$

Now we consider two kinds of parametrisation which are equivalent to introducing the constraints $r = r_0 = \text{constant}$ and $\theta = \theta_0 = \text{constant}$ respectively.

In the first case the solution for equation (4.5) is immediately given as

$$r_0(p_\theta) \approx a + \frac{1}{a} \left(\frac{p_\theta^2}{2Km} \right)^{1/3} \quad (4.7)$$

if we assume that the condition

$$\left(\frac{p_\theta^2}{2Km} \right)^{1/3} \ll a^2 \quad (4.8)$$

holds, which expresses the validity of the adiabatic approximation in this case. We have used $p_\theta = mr_0^2 \dot{\theta} = \text{constant}$.

This type of motion can be characterised by the parameters (θ, p_θ) , the functions of the parametrisation being given by

$$g_1 = r_0 \cos \theta \quad g_2 = r_0 \sin \theta \quad f_1 = -\sin \theta / r_0 \quad f_2 = \cos \theta / r_0. \quad (4.9)$$

We may also write the Hamiltonian

$$\mathcal{H} = \frac{p_\theta^2}{2J} + W_0 \quad (4.10)$$

where

$$J = mr_0^2(p_\theta) \quad (4.11)$$

and

$$W_0 = V(r_0(p_\theta)). \quad (4.12)$$

We shall prove now that equation (4.7) corresponds to a stable solution.

Indeed, if the circular trajectory of radius r_0 is slightly perturbed, giving $r = r_0 + \delta r$, we can write the equation of the time evolution of the deviation δr as

$$\delta\ddot{r} + \left[\frac{3p_\theta^2}{m^2 r_0^4} + \frac{6K}{m} (r_0 - a)^2 \right] \delta r = \text{constant}. \quad (4.13)$$

This is the familiar equation of a harmonic oscillator, and, as the quantity in square brackets is positive, it leads to an oscillating solution. This means that, if the particle is

slightly deviated from its initial path, there is a restoring force that keeps it moving around the solution $r = r_0$ for any time. This fact enables us to conclude that the initial solution is stable, and so the surface defined by the parametrisation used here is also stable.

Let us now analyse the second case. Equation (4.5) can be given in the form

$$m\ddot{r} + 2K(r - a)^3 = 0 \quad (4.14)$$

with a solution that we assume to be known $r(t) = \varphi(C_1, C_2, t)$.

We must again make some reference to the functions of the parametrisation

$$g_1 = r \cos \theta_0 \quad g_2 = r \sin \theta_0 \quad f_1 = \cos \theta_0 \quad f_2 = \sin \theta_0 \quad (4.15)$$

and to the Hamiltonian

$$\mathcal{H} = \frac{p_r^2}{2m} + \frac{K}{2}(r - a)^4 \quad (4.16)$$

where r and p_r are now the useful parameters to characterise the motion.

It is easily shown that in both cases the functions of the parametrisation are consistent with the conditions necessary to define the 'collective' degrees of freedom given by equations (I), (II) and (III).

As before, we can say something about the stability of the solution obtained in that case if the particle is slightly perturbed and we investigate its behaviour as a consequence of that perturbation.

We shall write now a new set of functions to characterise the motion of the particle

$$\begin{aligned} r(t) &= \varphi(C_1, C_2, t) + \delta r & p_r(t) &= m\dot{\varphi}(C_1, C_2, t) + \delta p_r \\ \theta(t) &= \theta_0 + \delta\theta & p_\theta(t) &= \delta p_\theta \end{aligned} \quad (4.17)$$

and the time evolution of the system must obey the Hamilton equations as usual.

Thus we can write

$$\delta\dot{\theta} = \dot{\theta} \approx \frac{\delta p_\theta}{m(\varphi(C_1, C_2, t))^2} \quad (4.18)$$

$$\delta\dot{p}_\theta = 0 \quad (4.19)$$

which means that the function $\delta\theta(t)$ is always an increasing or decreasing function of time.

There is no neighbourhood of the initial solution where the motion of the particle remains for any time. The solution is not stable, and so we can say nothing about the surface that this parametrisation defines.

As a conclusion of this simple example we have presented, we can now elaborate a criterion of collectivity. In fact we must remark that only the variables (θ, p_θ) and the corresponding functions $(g_i(\theta, p_\theta), f_i(\theta, p_\theta))$ are associated with a stable solution for the motion of the particle.

This suggests, as we have already inferred, that the natural criterion for the definition of the 'collective' path should be that of the stability of the corresponding solution.

Finally we consider a two-dimensional liquid drop, and we define for this model the collective mass parameter and the collective potential.

We deform the liquid drop in such a way that the displacement vector is

$$\mathbf{u}(x, y) = \alpha(x\hat{i} - y\hat{j}) \tag{4.20}$$

and the momentum vector is

$$\mathbf{p}(x, y) = \pi K(x\hat{i} - y\hat{j}) \tag{4.21}$$

where α and π are canonically conjugate and K is a constant which satisfies a normalisation condition equivalent to equation (I), i.e.

$$\iint dx dy K(x^2 + y^2)\sigma = 1 \tag{4.22}$$

σ being the surface density of the particles.

From equation (4.22) we can write

$$K = 2/\Pi a^4 \sigma \tag{4.23}$$

where a is the radius of the liquid drop.

Equations (4.20) and (4.21) related by the condition (4.22) allow the Lagrangian to be written in the form

$$\mathcal{L}(\alpha, \pi) = \pi\dot{\alpha} - \mathcal{H}(\alpha, \pi). \tag{4.24}$$

Looking for the function $\mathcal{H}(\alpha, \pi)$, we first evaluate the kinetic energy

$$\mathcal{T}(\pi) = \sum_i \frac{p_i^2}{2m} = \iint dx dy \frac{\pi^2 K^2}{2m} (x^2 + y^2)\sigma = \frac{\pi^2}{\Pi m a^4 \sigma}. \tag{4.25}$$

It is natural to assume that the range of the two-body interaction is of the order of the single-particle radius. Under such conditions, the potential energy will be proportional to the surface extension.

In fact, if each particle interacts with its nearest neighbours, the potential will be

$$V = \sum_{i < j} v_{ij} = \frac{1}{2} n v N - \frac{1}{2} \frac{n}{2} v N' \tag{4.26}$$

where v is the interaction between any two particles, N is the total number of particles, n is the number of particles which interact with each one of them, and N' is the number of particles that surround the drop. The second term compensates for the fact that the first term overestimates the interactions of the surface particles which interact only with $n/2$ particles instead of n .

If we suppose that each particle has a radius equal to r_0 , N' can be given by

$$N' = S/2r_0 \tag{4.27}$$

where S is the length of the ellipse which results from the deformation of the drop and r_0 is defined by the equation

$$A = N\Pi r_0^2. \tag{4.28}$$

A represents the area of the liquid drop.

If the deformation is small enough we can approximate S to the expression

$$S = 2\Pi(a + \alpha^2 a/4). \tag{4.29}$$

Then, dropping the constant term in equation (4.26), we can write

$$V = -C\alpha^2 \quad (4.30)$$

where C is given by

$$C = \Pi av/4r_0 \quad (4.31)$$

when we use equations (4.27) and (4.29) and choose, for this model, $n = 4$.

Finally, we can define a 'collective' mass parameter and a 'collective' potential given respectively by

$$\mathcal{M} = \frac{1}{2}m\sigma a^4\Pi \quad (4.32)$$

and

$$\mathcal{V}(\alpha) = -C\alpha^2. \quad (4.33)$$

We can also derive, for the frequency of the oscillation of the drop, the expression

$$\omega = (v/2ma^3\sigma r_0)^{1/2}. \quad (4.34)$$

Remark. The symbol Π means 3.1415.

5. Conclusion

In the present paper we have shown that, by using a parametrisation of the coordinates, one is able to describe, under some circumstances, the dynamical features of a many-body system in terms of two (or few) parameters, without specifying the individual coordinates and velocities.

We have assumed the adiabatic approximation and followed a classical formulation in order to get the classical equivalent to a 'collective' Hamiltonian. We have obtained necessary conditions which the parametrisation functions should obey. We have emphasised the formal equivalence between these and Villar's corresponding equations. This formalism has been applied to simple model systems. We have used, in the first case, two different parametrisations which are formally equivalent to the introduction of constraints, allowing us to reduce the number of degrees of freedom to those necessary for the study of a particular behaviour of the system.

A careful analysis of the different kinds of solution has allowed us to formulate a sufficient condition of collectivity to complete the necessary conditions referred to before.

In conclusion, it seems that the first example enables us to set up an analogy between the introduction of certain types of constraints in classical mechanics and the definition of the collective degrees of freedom in a quantal many-body problem. It also suggests that the existence of great stability trajectories can be associated with the collective evolution of a many-particle system.

With the second model we have in mind to illustrate a more realistic application of our basic ideas.

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